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ORTHOTROPIC BENDING PROPERTIES OF FIBERBOARD

Earl C. Steeves

Army Natick Development Center
Natick, Massachusetts

November 1975

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UNITED STATES ARMY
NATICK DEVELOPMENT CENTER
NATICK, MASSACHUSETTS 01760

Aero-Mechanical Engineering Laboratory

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Bend Properties	Recording Apparatus	Adhesive Bonding	Containers
	Recording Equipment	Bonding Strength	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An experimental procedure for the determination of the orthotropic bending stiffness of fiberboard is described, including the loading apparatus, strain gauges utilized, recording equipment and data reduction procedure. Results obtained using the described procedure are presented for V3C fiberboard. These results indicate that the assumption of orthotropic behavior is valid, that this grade of corrugated fiberboard is least flexible in the direction perpendicular to the corrugations and that the stiffness properties of the fiberboard are not significantly degraded by the attachment of adhesive bonded resistance strain gauges.

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PREFACE

The results presented in this paper were developed during an investigation of the behavior of package and container structures under load. This work was initiated under Task 1J662708D552 05/017, Establishment of Design Criteria for Containers, and carried forward under Task 1E662703A090–04; Design, Analysis and Optimization of Structures; through the AMC Computer-Aided Design and Engineering Program.

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INTRODUCTION

The rational design of packages and containers requires the ability to predict the deformation and stress of these structures resulting from the loads experienced during their service life. One approach to achieving this goal is the application of the large body of theory that has been developed in structural mechanics. In order to utilize this theory, however, it is necessary to have the appropriate stress-strain law for the material used in the construction of the package or container. One of the most commonly used packaging materials is corrugated fiberboard which is generally used in the form of thin sheets and frequently acts as a bending member. Thus the theory of thin plates can be utilized as a deformation and stress prediction tool. This theory requires that the stress-strain law be in the form of moment-curvature relations. Examination of the geometry of corrugated fiberboard suggests that it cannot be treated as an isotropic material; however, it does possess two orthogonal axes of symmetry which suggests the orthotropic model that has been adopted.

The bending behavior of orthotropic materials is well understood but no explicit model for corrugated fiberboard is presently known and the purpose of this report is to present a procedure for obtaining such a model and to give an example of the use of the procedure for a common grade of fiberboard. There is nothing in the procedure, however, that restricts its use to fiberboard. The procedure used to transform the experimental data into the orthotropic bending law is an adaptation of a procedure for the general orthotropic stress-strain law presented by Lempriere.*

*1. B.M. Lempriere, Uniaxially Loading of Orthotropic Materials, AIAA Journal, Vol. 6, No. 2, February 1968.

EXPERIMENTAL PROCEDURE

A procedure for the determination of the stress-strain law of the more complex materials such as those having orthotropic symmetry requires techniques for the acquisition of data regarding both the stress and strain and mathematical formulas for the transformation of this data into a stress-strain law. These acquisiton techniques and mathematical formulas are described in this section of the report.

Experimental Apparatus

The determination of the bending properties requires the simultaneous measurement of both the moment and the curvature. For the measurement of the curvature which is a measure of strain, it is recalled that the change in curvature of a bending member can be expressed as

$$\kappa'_{11} = \frac{\partial^2 w}{\partial x_1'^2} \tag{1}$$

where, as shown in Figure 1, W is the transverse or normal displacement, x_1' is the coordinate direction in which the curvature is being measured and κ'_{11} is the curvature. The strain at any point in the cross section is given as

$$\epsilon'_{11} = -x_3' \frac{\partial^2 w}{\partial x_1'^2} \tag{2a}$$

or

$$\epsilon'_{11} = -x_3'\kappa'_{11} \tag{2b}$$

where ϵ'_{11} is the direct strain associated with the x_1' coordinate direction and x_3' is the independent coordinate in the transverse direction, as shown in Figure 1. If the strain, ϵ'_{11}, is measured on the specimen surface defined by $x_3' = h/2$ then the curvature can be computed using equation (2). A better experimental procedure consists of making strain measurements on both surfaces of the specimen and computing the curvature from the relationship

$$\kappa'_{11} = \frac{\epsilon'_{11}(-h/2) - \epsilon'_{11}(h/2)}{h} \tag{3}$$

In pure bending it is expected that ϵ'_{11} (h/2) and ϵ'_{11} (−h/2) will be equal and thus give the same result as the single-measurement procedure. However, the redundant procedure provides a check on the experimental data through the expected equality. In dealing

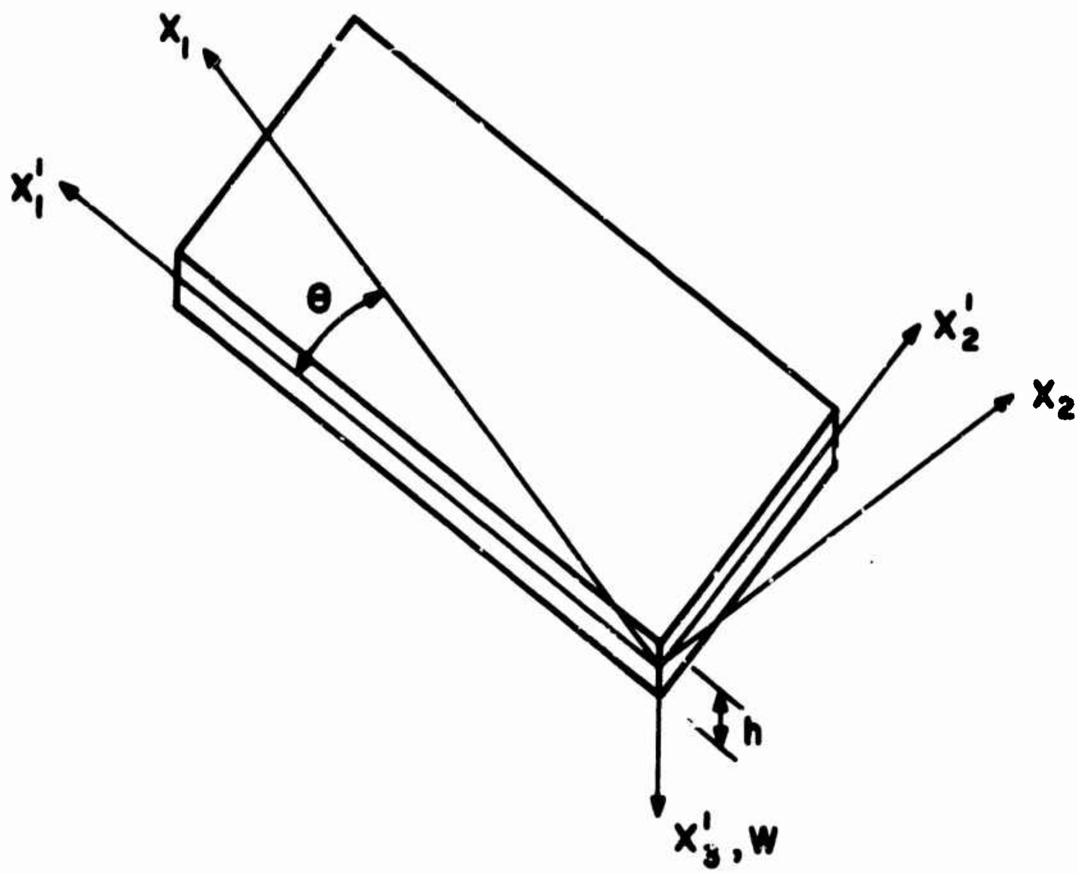

Figure 1. Definition of Coordinates

with orthotropic materials it is necessary to measure the curvature in two orthogonal directions one of which coincides with the direction of the applied moment. This is accomplished by the use of rosette strain gages to determine $\epsilon_{2'2}'(h/2)$ and $\epsilon_{2'2}'(-h/2)$ along with $\epsilon_{1'1}'(h/2)$ and $\epsilon_{1'1}'(-h/2)$. The curvature $\kappa_{2'2}'$ associated with x_2' coordinate line is determined by

$$\kappa_{2'2}' = \frac{\epsilon_{2'2}'(-h/2) - \epsilon_{2'2}'(h/2)}{h} \tag{4}$$

All of the strain measurements were carried out using resistance wire strain gages on paper backing. The bonding of the gages to the fiberboard specimen was accomplished with Eastman 910 adhesive.

In addition, a simultaneous measurement must be taken of the moment existing at the point where the curvatures are being measured. This moment must be determined directly as opposed to being inferred from a strain measurement. This requires a simple loading device which is statically determinant. Such a device is illustrated in Figure 2 and is typical of a class in general use for various bending tests. The usefulness of this loading device results from the fact that the moment is constant throughout the test section and this greatly relieves the necessity for exact positioning of strain gages on the specimen and of the specimen in the loading fixture. The magnitude of the moment in the test section is given in terms of the applied force, P, as

$$M_{1'1}' = \tfrac{1}{2}P(d_2 - d_1) \tag{5}$$

For the fixture used in conjunction with the work being reported here, d_1 and d_2 are respectively 0.0381m and 0.0762m. The orientation of the specimen and loading device in the x_1' coordinate direction is shown in Figure 2.

For completeness, a brief description of the remainder of the test and recording equipment used follows. The load application and measurement was accomplished through the use of a table model Instron testing machine. The strain gage bridge circuits were excited with a d.c. power supply through a bridge balancing unit. The voltage resulting from the imbalance in the bridge circuit due to strain was recorded on an oscillograph. The output from the Instron load weighing system was also recorded on this oscillograph; however, a buffer amplifier was required to provide an impedance match between the oscillograph and the load weighing electronics associated with the Instron. A photograph of this test apparatus is shown in Figure 3. In this figure the wiring of the strain gauges to recording equipment are not shown.

With the data obtained using this appartus it is possible to construct plots of the changes in curvatures $\kappa_{1'1}'$ and $\kappa_{2'2}'$ as functions of the applied moment $M_{1'1}'$; typical results

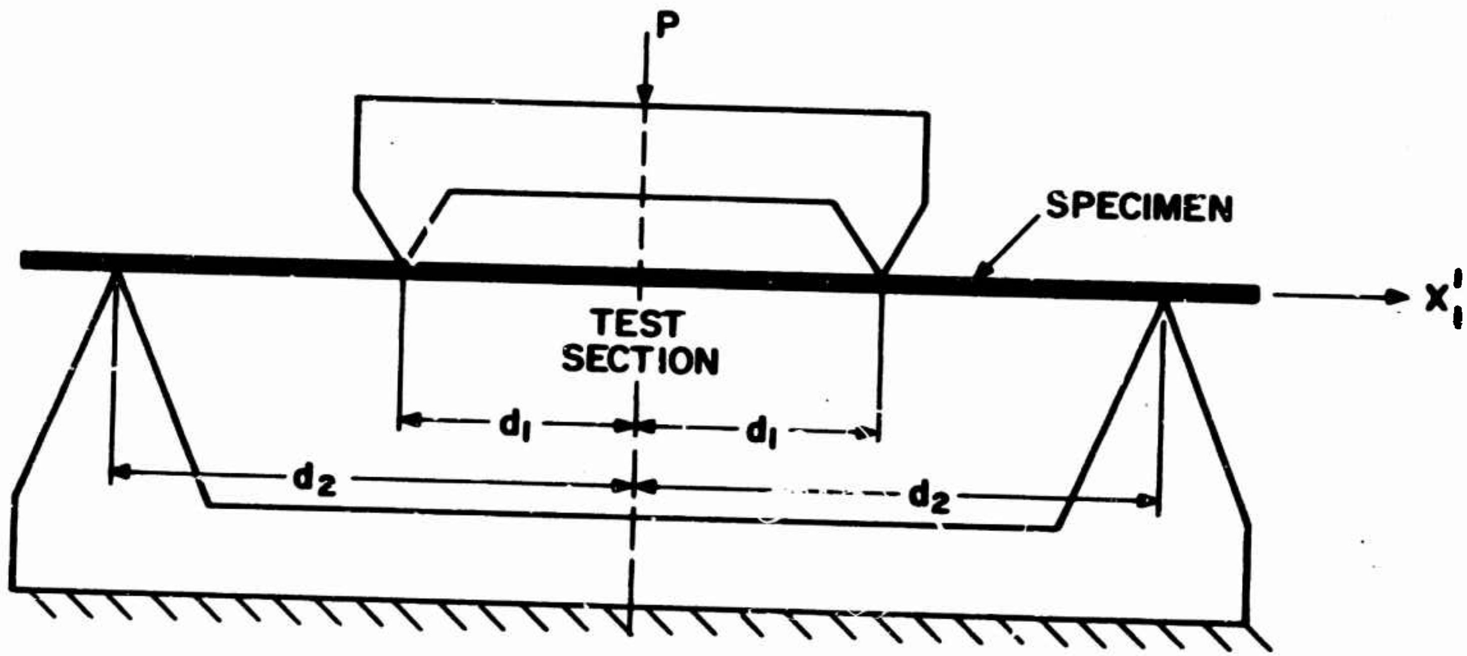

Figure 2. Loading Fixture

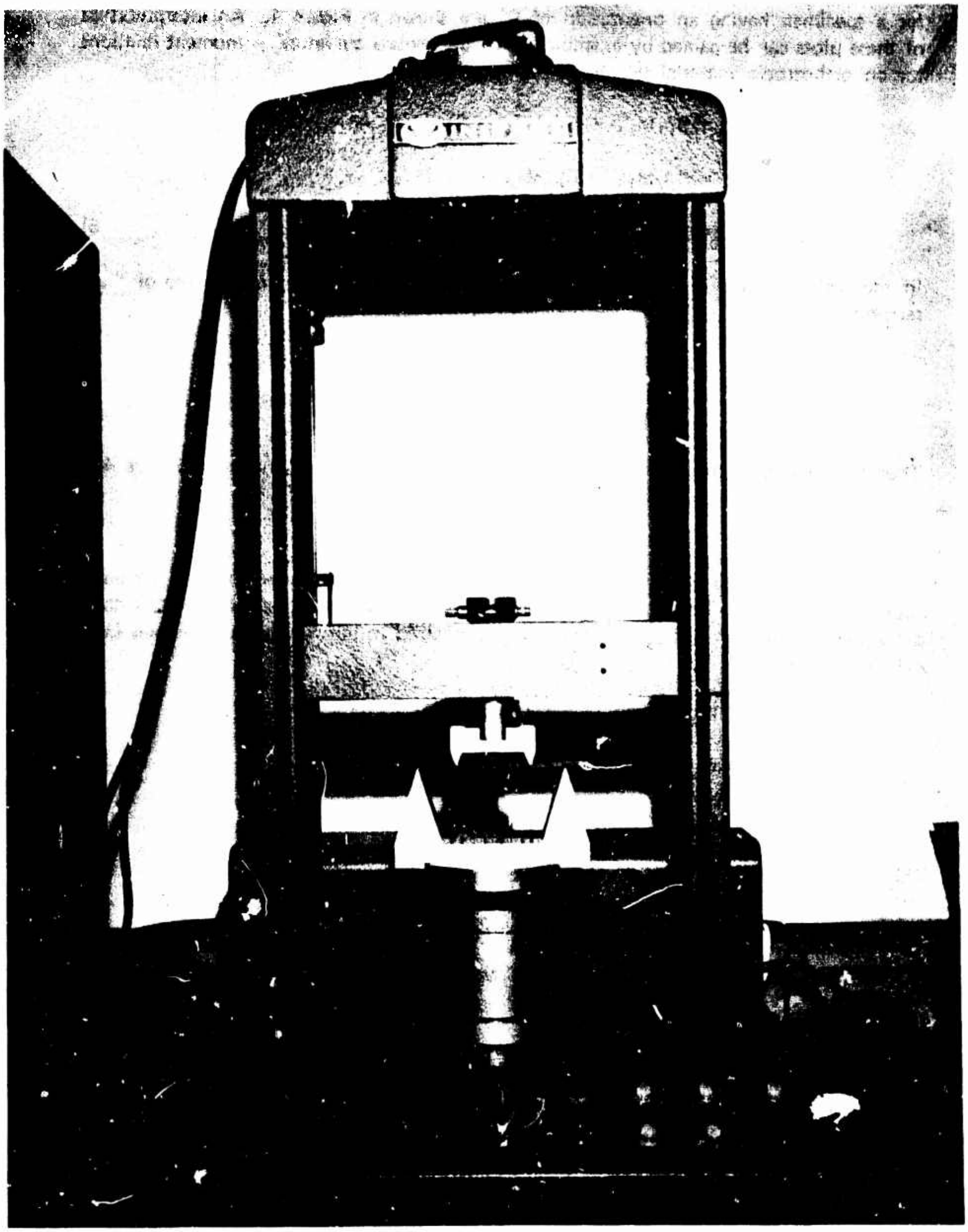

Figure 3. Photograph of Test Apparatus

for a specimen having an orientation of 0° are shown in Figure 4. An interpretation of these plots can be gained by examination of the general curvature — moment relations for an orthotropic material which can be stated as

$$\kappa'_{11} = S'_{11}M'_{11} + S'_{12}M'_{22} + S'_{13}M'_{12}$$

$$\kappa'_{22} = S'_{12}M'_{11} + S'_{22}M'_{22} + S'_{23}M'_{12}$$

$$\kappa'_{12} = S'_{13}M'_{11} + S'_{23}M'_{22} + S'_{33}M'_{12} \qquad (6)$$

In the test described M'_{11} is the only nonvanishing moment so the first two of these relations become

$$\kappa'_{11} = S'_{11}M'_{11}$$

$$\kappa'_{22} = S'_{12}M'_{11} \qquad (7)$$

Thus S'_{11} and S'_{12} are the slopes of the moment-curvature plots shown in Figure 4.

Data Reduction

In the previous section the test procedure was described and the form of the data obtained was given. The purpose of the present section is to describe how this data is transformed into an orthotropic bending law referred to the principle coordinates (x_1, x_2) in the form:

$$M_{11} = D_{11}\kappa_{11} + D_{12}\kappa_{22}$$

$$M_{22} = D_{12}\kappa_{11} + D_{22}\kappa_{22} \qquad (8)$$

$$M_{12} = D_{33}\kappa_{12}$$

This expression can be inverted to give:

$$\kappa_{11} = S_{11}M_{11} + S_{12}M_{22}$$

$$\kappa_{22} = S_{12}M_{11} + S_{22}M_{22} \qquad (9)$$

$$\kappa_{12} = S_{33}M_{12}$$

which is the same form as equation (6), except that it is referred to the coordinates (x_1, x_2) which are the directions of orthotropic symmetry. Relative to these directions the coupling between the twisting, κ_{12}, and the bending moments M_{11} and M_{22} vanishes.

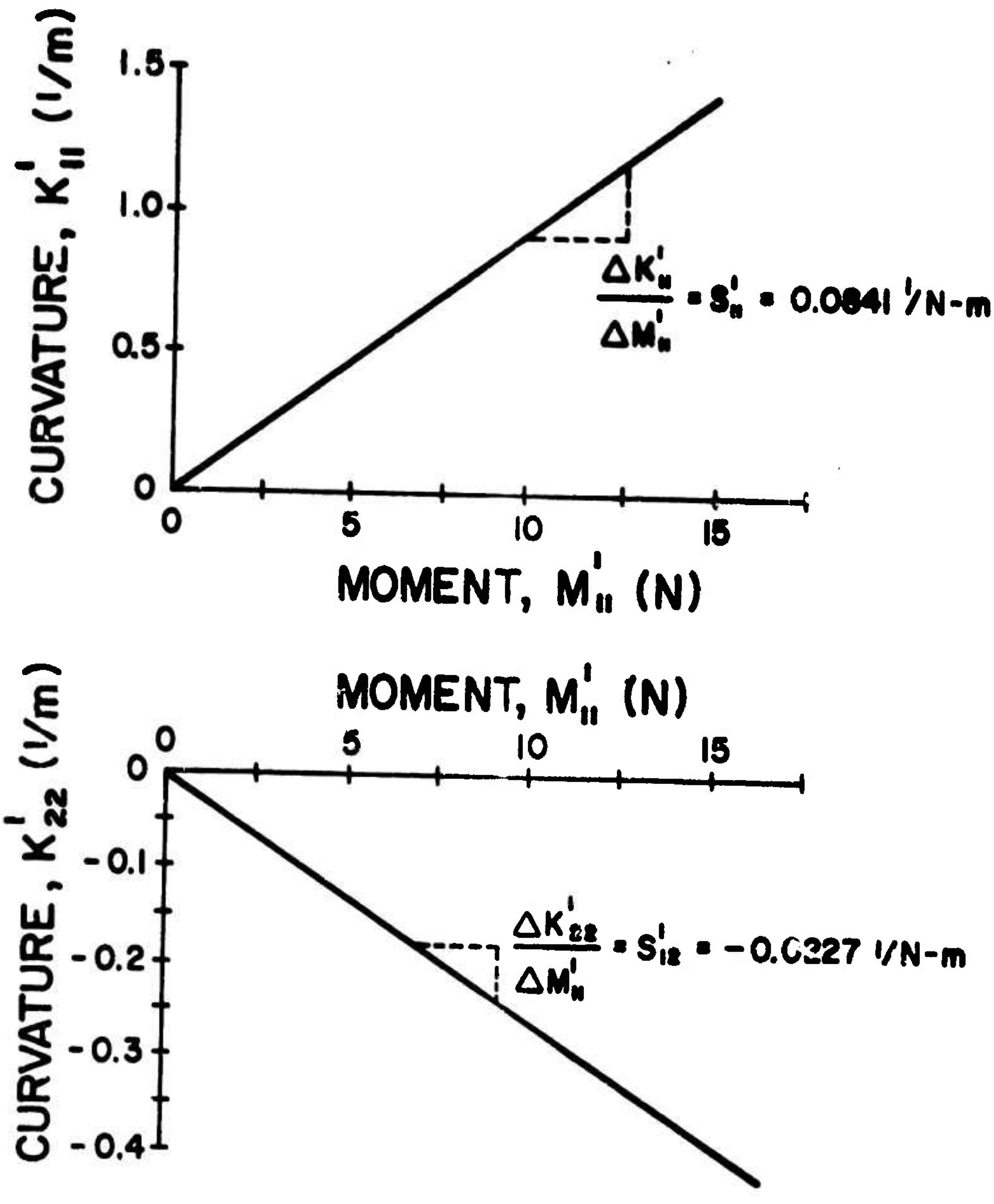

Figure 4. Moment Curvature Diagrams

If the parameters S_{ij} are determined then the parameters D_{ij} can be expressed in terms of them and this is the procedure that will be followed.

From the test of a specimen of orientation θ the parameters S'_{11} and S'_{12} for that orientation can be determined. These parameters can be expressed in terms of the parameters S_{ij}, referred to the principle coordinates (x_1, x_2), and θ, the angle of orientation of the specimen with respect to the principle coordinates, as

$$
\begin{aligned}
S'_{11} &= S_{11}\ \mathrm{Cos}^4(\theta) + 2S_{12}\mathrm{Cos}^2(\theta)\mathrm{Sin}^2(\theta) + S_{22}\mathrm{Sin}^4(\theta) \\
&\quad + 2S_{33}\ \mathrm{Cos}^2(\theta)\mathrm{Sin}^2(\theta) \\
S'_{12} &= S_{11}\mathrm{Sin}^2(\theta)\ \mathrm{Cos}^2(\theta) + S_{12}\ [\mathrm{Sin}^4(\theta) + \mathrm{Cos}^4(\theta)] \\
&\quad + S_{22}\mathrm{Sin}^2(\theta)\mathrm{Cos}^2(\theta) - 2S_{33}\mathrm{Cos}^2(\theta)\mathrm{Sin}^2(\theta)
\end{aligned}
\tag{10}
$$

Following the procedure given in reference 1, these expressions can be transformed to the following:

$$
\begin{aligned}
S'_{12} &= (S_{11} + S_{22} - 2S_{33} - 2S_{12})\ \mathrm{Sin}^2(\theta)\mathrm{Cos}^2(\theta) + S_{12} \\
S'_{11} + S'_{12} &= (S_{22} + S_{12}) + (S_{11} - S_{22})\ \mathrm{Cos}^2(\theta)
\end{aligned}
\tag{11}
$$

Thus we have S'_{12} expressed as a linear function of $\mathrm{Sin}^2(\theta)\mathrm{Cos}^2(\theta)$ and the sum $S'_{11} + S'_{12}$ expressed as a linear function of $\mathrm{Cos}^2(\theta)$. If experimental results S'_{11} and S'_{12} are obtained for a number of values of θ, or from specimens having a number of orientations, then plots of S'_{12} vs $\mathrm{Sin}^2(\theta)\mathrm{Cos}^2(\theta)$ and $S'_{11} + S'_{12}$ vs $\mathrm{Cos}^2(\theta)$ can be made and a least squares linear function fit to the data. Having these linear functions with slopes m_1 and m_2 and intercepts y_1 and y_2 for the S'_{12} and $(S'_{11} + S'_{12})$ functions the following relations can be established:

$$
\begin{aligned}
S_{11} + S_{22} - 2S_{33} - 2S_{12} &= m_1 \\
S_{12} &= y_1 \\
S_{11} - S_{22} &= m_2 \\
S_{22} + S_{12} &= y_2
\end{aligned}
\tag{12}
$$

This gives four equations in the four unknowns, S_{ij}, which have the following solution

$$
\begin{aligned}
S_{12} &= y_1 \\
S_{22} &= y_2 - y_1 \\
S_{11} &= m_2 + y_2 - y_1 \\
S_{33} &= \frac{1}{2}(m_2 - m_1 + 2y_2 - 4y_1)
\end{aligned}
\tag{13}
$$

With the S_{ij} determined, the D_{ij} can be computed by inversion of equation (9). The details of carrying out this procedure are illustrated with actual data from fiberboard specimens in the next section.

RESULTS

The purpose of this section is to illustrate the procedure described above with some actual experimental data obtained using the apparatus previously described. The result will be the specification of the orthotropic moment-curvature law for V3C corrugated fiberboard.

The moment-curvature data for several values of the orientation angle is presented in Table I. It will be noted several sets of data were obtained for each value of θ and that in addition to the data, Table I includes the values of $\text{Sin}^2(\theta)\text{Cos}^2(\theta)$ and $\text{Cos}^2(\theta)$ which are needed in carrying out the data reduction. The parameters $(S'_{11} + S'_{12})$ and S'_{12} are presented graphically as functions of $\text{Cos}^2\theta$ and $\text{Sin}^2\theta\text{Cos}^2\theta$ respectively in Figure 5 along with the least squares linear fit of the data. The equations for these linear functions are

$$(S'_{11} + S'_{12}) = 0.02659 + 0.03037 \ \text{Cos}^2(\theta) \tag{14}$$

$$S'_{12} = -0.01799 - 0.006715 \ \text{Sin}^2(\theta)\text{Cos}^2(\theta)$$

Thus the values of the slopes and intercepts are

$$\begin{aligned} y_1 &= -0.01799 \\ y_2 &= 0.02659 \\ m_1 &= -0.006715 \\ m_2 &= 0.03037 \end{aligned} \tag{15}$$

substitution of these values into equation (13) gives

$$\begin{aligned} S_{12} &= -0.01799 \ \text{1/N-m} \\ S_{22} &= 0.04458 \ \text{1/N-m} \\ S_{11} &= 0.07495 \ \text{1/N-m} \\ S_{33} &= 0.08111 \ \text{1/N-m} \end{aligned} \tag{16}$$

TABLE I

MOMENT—CURVATURE PROPERTIES AS A FUNCTION OF SPECIMEN ORIENTATION

θ Deg.	$\Delta M'_{11}$ N	$\Delta \kappa'_{11}$ 1/m	$\Delta \kappa'_{22}$ 1/m	S'_{11} 1/N-m	S'_{12} 1/N-m	$S'_{11}+S'_{12}$ 1/N-m	$\cos^2\theta \sin^2\theta$	$\cos^2\theta$
0	14.99	1.26	−0.34	8.41×10^{-2}	-2.27×10^{-2}	6.14×10^{-2}	0.0	1.0
	7.87	0.53	−0.17	6.73	−2.16	4.57		
15	16.06	0.89	−0.23	5.54	−1.43	4.11	0.0625	0.933
	11.30	0.84	−0.24	7.34	−2.12	5.22		
30	11.48	0.91	−0.27	7.93	−2.35	5.58	0.1875	0.75
	7.52	0.69	−0.19	9.17	−2.53	6.64		
45	12.63	0.76	−0.23	6.02	−1.82	4.20	0.25	0.5
	13.83	0.79	−0.33	5.71	−2.39	3.32		
	9.56	0.71	−0.17	7.43	−1.78	5.65		
60	14.81	0.85	−0.24	5.74	−1.62	4.12	0.1875	0.25
	12.10	0.62	−0.17	5.12	−1.40	3.72		
	12.72	0.61	−0.21	4.79	−1.65	3.14		
75	15.03	0.69	−0.26	4.59	−1.73	2.86	0.0625	0.0669
	8.14	0.35	−0.16	4.30	−1.96	2.34		
90	14.06	0.61	−0.23	4.34	−1.64	2.70	0.0	0.0
	8.85	0.27	−0.11	3.05	−1.24	1.81		

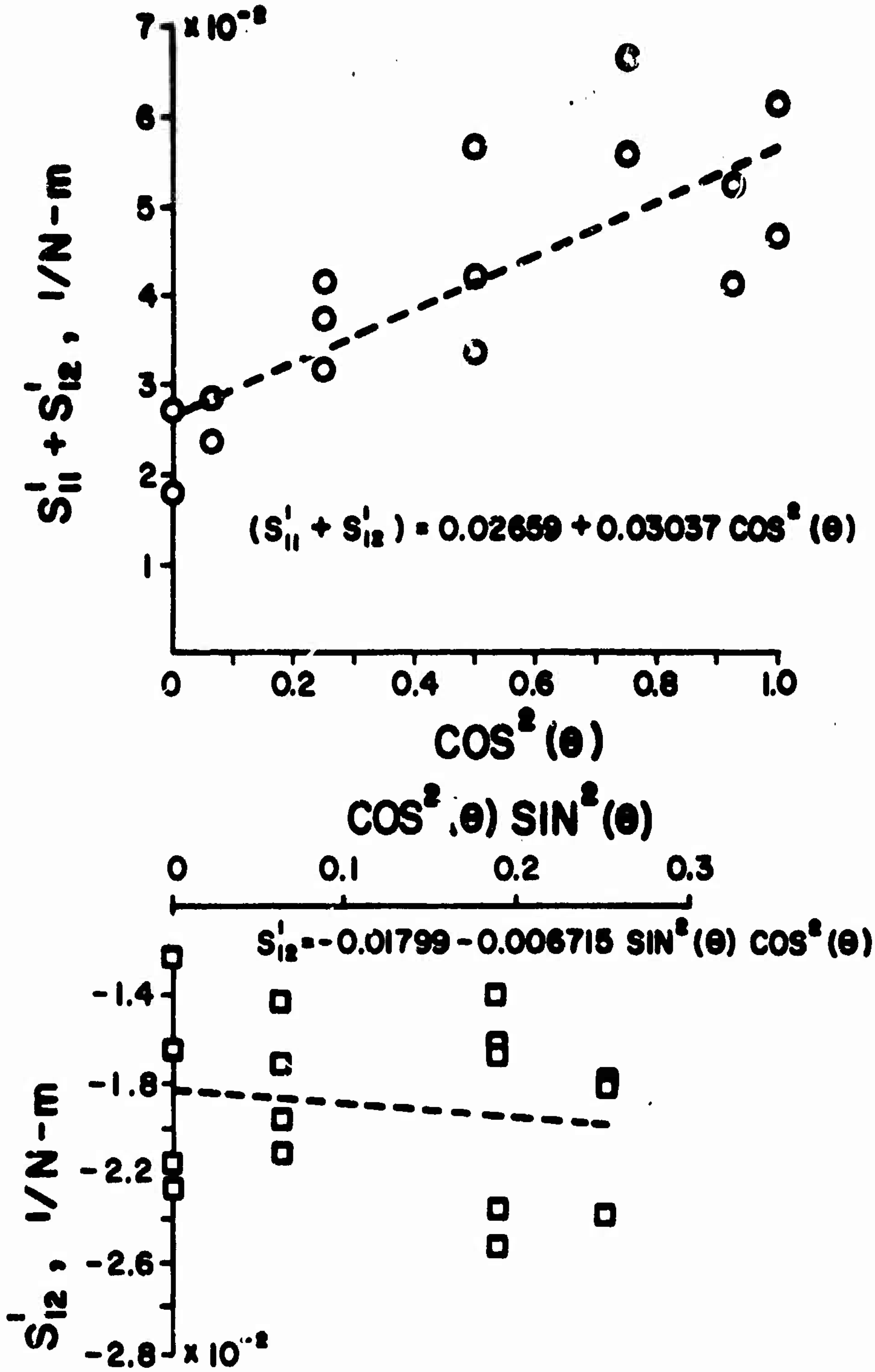

Figure 5. Plc.s of Bending Flexibilities Used in Data Reduction

The stiffness parameters D_{ij} which are the most useful for theoretical work are given in terms of the S_{ij} as:

$$
\begin{aligned}
D_{11} &= S_{22}/S_{11}\ S_{22} - S^2{}_{12} \\
D_{12} &= -S_{12}/S_{11}\ S_{22} - S^2{}_{12} \\
D_{22} &= S_{11}/S_{11}\ S_{22} - S^2{}_{12} \\
D_{33} &= 1/S_{33}
\end{aligned}
\qquad (17)
$$

Using the values given in (16), the following are the orthotropic stiffness parameters for V3C fiberboard

$$
\begin{aligned}
D_{11} &= 14.77 \text{ N-m} \\
D_{12} &= 5.96 \text{ N-m} \\
D_{22} &= 24.84 \text{ N-m} \\
D_{33} &= 12.33 \text{ N-m}
\end{aligned}
\qquad (18)
$$

This provides a complete specification of the bending stiffness through the D_{ij} and the bending compliance through the S_{ij}. The D_{ij} and S_{ij} are the orthotropic constants relative to the principal directions and are used to compute the compliance and stiffness parameters relative to any other directions according to transformations given by Hearmon*.

The behavior of the compliances S'_{11} and S'_{12} as a function of orientation is given by equations (10). This behavior for V3C fiberboard based on the results given in equation (16) is shown in Figure 6 along with the experimentally determined values of S'_{11} and S'_{12}. Examination of these results indicates that the orthotropic model specified by the constants S_{ij} of equation (16) is in good agreement with the experimental results from which it was derived, thus justifying the assumption of the orthotropic model. Note that the orientation of 0° corresponds to the situation in which the corrugations of the fiberboard are parallel to the x'_1 axis. It is expected that in this orientation the corrugations would be most effective and the fiberboard would be least compliant or flexible. However, the data presented in Figure 6 indicate that this is not the case and that the least flexible orientation is that corresponding to an angle of 90°. One possible explanation of this behavior is that the corrugations do not provide any significant inherent stiffness but merely act as separators for the face sheets and the orthotropic behavior observed here is due solely to the behavior of the face sheets which are assembled with their most flexible orientation parallel to the corrugations.

*2. R.F.S. Hearmon, An Introduction to Applied Anisotropic Elasticity; Oxford University Press; 1961.

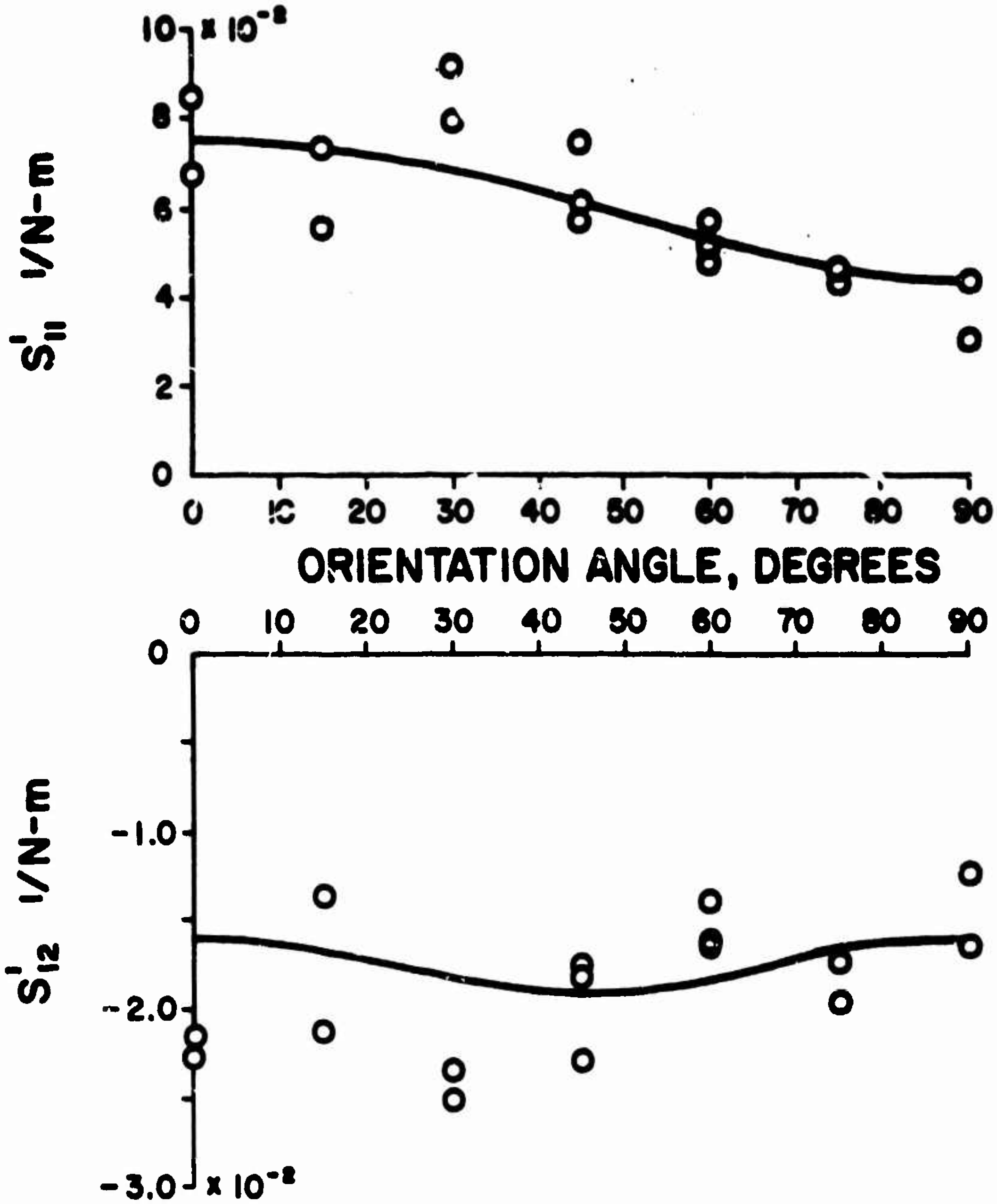

Figure 6. Behavior of V_3C Fiberboard Compliances as a Function of Orientation

The question of possible stiffening of the fiberboard by the attachment of the strain gages was addressed by making stiffness measurements of a number of specimens both with and without strain gages. The loading apparatus used was that shown in Figure 2 with load-deflection curves obtained directly from the Instron testing machine. The slopes of these load deflection curves which are a measure of stiffness are given in Table II for specimens with and without strain gages of several orientations. Examination of this data reveals that no clear indication of stiffening is evident. In two cases, 45° and 60°, the specimens with strain gages are on the order of 10% stiffer. The remainder of the samples are divided between cases with the specimen having strain gages being stiffer and less stiff. The increase or decrease in all these cases is on the order of 2 to 3%. Thus the case for stiffening due to strain gauge bonding is not compelling. In addition, this data further confirms that, as discussed above, the stiffest or least flexible orientation corresponds to an orientation of 90°.

TABLE II

COMPARISON OF SPECIMEN STIFFNESS
WITH AND WITHOUT STRAIN GAGES

Orientation Angle (°)	Stiffness – N/m	
	With Strain Gages	Without Strain Gages
0	8.19×10^3	8.41×10^3
15	8.14	7.95
30	8.76	8.55
45	9.99	8.98
60	10.28	9.47
75	9.63	9.82
90	11.42	10.61

CONCLUDING REMARKS

An experimental procedure and an associated data reduction scheme for the determination of the orthotropic bending properties of fiberboard has been developed. Results from an application of the procedure to V3C corrugated fiberboard revealed the following:

1. The assumption of orthotropic behavior is valid.

2. The least flexible orientation of V3C fiberboard does not correspond to the direction parallel with the corrugations.

3. The use of adhesive bonded resistance wire strain gages on fiberboard does not result in sufficient stiffening of the fiberboard to cause degradation of the results.

Although the application here was to fiberboard there is nothing in the procedure to limit it to that material.

LIST OF SYMBOLS

D_{ij}	Orthotropic Stiffness Constants
d_1, d_2	Test Fixture Dimensions
h	Thickness of Specimen
M_{11}, M_{22}, M_{12}	Moment Components
m_1, m_2	Slopes of Linear Functions
P	Applied Force
S_{ij}	Orthotropic Compliance Constants
W	Normal Displacement Component
x_1, x_2, x_3	Coordinates Corresponding to the Axes of Orthotropic Symmetry
x_1', x_2', x_3'	Coordinates Defining Specimen Orientation
y_1, y_2	Intercepts of Linear Functions
$\epsilon_{11}, \epsilon_{22}, \epsilon_{12}$	Strain Components
θ	Angle Specifying the Specimen Orientation
$\kappa_{11}, \kappa_{22}, \kappa_{12}$	Curvature Components
$(\ )'$	Denotes Parameter Referred to the x_1', x_2', x_3' Coordinates